# Chiral phase transition in an expanding quark antiquark plasma

**Abdellatif Abada**[*†] **and Jörg Aichelin**[‡]

Laboratoire de Physique Nucléaire, CNRS/IN2P3 Université de Nantes,

2, Rue de la Houssinière, 44072 Nantes Cedex 03, France

**Abstract**

We investigate the time evolution of a quark antiquark plasma by solving numerically the relativistic transport equations derived on the Hartree level from the Nambu-Jona-Lasinio model. We find that the phase transition in the expanding quark antiquark plasma is different as compared to that in a static plasma. The expansion competes with the transition and finally quark droplets will be formed which subsequently hadronizes. These findings raise the question whether static thermal models can make at all any prediction about signals of that transition .

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[*]email : "ABADA@ptdec1.tphys.physik.uni-tuebingen.de"

[†]New address: Institute for Theoretical Physics, Tübingen University, Auf der Morgenstelle 14, D-72076 Tübingen

[‡]email : "AICHELIN@nanvs2.in2p3.fr"

The search for the Quark Gluon Plasma (QGP) is one of the challenges of present day nuclear physics. It is the driving motivation of the lead beam experiments which started this year at CERN. There is first of all the question whether this phase transition can indeed be achieved with accelerated nuclei. The even bigger challenge is, however, to investigate which signals bear witness to the existence of the plasma, i.e., cannot be created by the dense hadronic medium which is present after the QGP has ceased to exist and the quarks are again bound in the hadrons which will finally be detected.

Up to now almost all investigations have tacitly assumed that the plasma is contained in a box whose boundaries can be moved that slowly that the systems is thermally equilibrated. Then, knowing the equation of state, the thermodynamical variables are sufficient to describe the transition to the hadronic world (for a review of the thermal scenario we refer to [1]). The reality, however, is most probably quite opposite. We have a gas of almost massless quarks at a temperature of more then 100 MeV and nothing which confines the plasma to a certain space region. Hence the system expands and we are confronted with the not yet investigated question how the phase transition takes place in the expanding non thermal system.

This question cannot be addressed in the framework of thermodynamics but requires a transport theory. In principle a transport theory may be derived from each Lagrangian and consequently also from the QCD Lagrangian. The attempts which have been made, however, do not come even close to an equation which can be used for numerical investigations [2]. Therefore one has to rely on phenomenological Lagrangians. One of these is the Nambu-Jona-Lasinio Lagrangian (NJL)[3]. In this Lagrangian all gluonic degrees of freedom are integrated out. The interaction between the quarks, however, is quite reliable modeled, giving the masses of the known hadrons with an acceptable precision. Its drawback is that it has no confinement, i.e., cannot describe properly the hadronization. Despite of this fact, this Lagrangian seems to be appropriate to start with the investigation of the question how a phase transition takes place in an expanding plasma. The results can be compared with that of the string models which recently also conjectured that mini plasmas have to be formed during the expansion of the system [4]. There is no doubt that this approach can be regarded only as a first step towards the understanding of the expansion of a quark gluon plasma.

By using the loop expansion approach of the two-point source connected generating functional [5] of the NJL Lagrangian, Zhang and Wilets [6] derived a microscopic transport theory to describe the chiral dynamics in high energy heavy-ion collisions. This transport theory which describes the time evolution of the Green's function is based on the the closed time-path Green's functions formalism [7]. At this level it seems, however, not yet possible to perform calculations. If one assumes that the dissipation parts of the Green's functions are small compared to the dispersive parts (the so-called quasiparticle

limit), the quasiparticle energy spectra of quarks and mesons are obtained from the equations giving the poles of the Green's functions, i.e., the zeros of the dispersive parts. Consequently, the Green's functions representation can be converted into phase space densities [6].

In this work we focus on the transport equation without collision terms. For the NJL model, the Vlasov equation for the quark density distribution $f(\mathbf{r},\mathbf{p},t)$ in the Hartree level reads as follow [6, 8]

$$\frac{\partial f}{\partial t}+\frac{\mathbf{p}}{E}.\boldsymbol{\nabla}_{\boldsymbol{r}}f-\boldsymbol{\nabla}_{\boldsymbol{r}}E.\boldsymbol{\nabla}_{\boldsymbol{p}}f=0 \tag{1}$$

where $E$ is the relativistic quark energy $\sqrt{\mathbf{p}^2+M_c^2}$ and $M_c$ is the constituent quark mass which is determined by the gap equation [6] :

$$M_c(\mathbf{r},t)=m+g^2M_c(\mathbf{r},t)\int_0^{\Lambda}\frac{\mathrm{d}^3p}{\sqrt{\mathbf{p}^2+M_c^2(\mathbf{r},t)}}(\frac{2N_cN_f}{(2\pi)^3}-f(\mathbf{r},\mathbf{p},t)-\tilde{f}(\mathbf{r},\mathbf{p},t)\ )\ . \tag{2}$$

In Eq.(2) $m$ is the quark mass ($m=m_u=m_d=4$ MeV), $N_c=3$ is the color number, $N_f=2$ is the flavor number, $g$ is the NJL four-point interaction coupling constant [3] and $\Lambda$ is the cutoff in momenta. In order to reproduce the pion mass and the pion decay constant the parameters $g$ and $\Lambda$ are fixed to 0.48 fm and 820 MeV respectively [6].

The solution of Eq.(1) is not trivial due to the dependence of the energy $E$ on the distribution $f$ via the constituent quark mass (c.f., Eq.(2)). For this reason we have to use some approximation scheme. In this work we use the test particles method which has been shown to be powerful in intermediate energy heavy ion collisions [9]. This approach consists to approximate the distribution function $f$ ($\tilde{f}$) by a crowd of "numerical" quarks (anti-quarks). Namely,

$$\begin{aligned} f(\mathbf{r},\mathbf{p},t)&=w\sum_{i=1}^{A}\delta^3(\mathbf{r}-\mathbf{r}_i(t))\ \delta^3(\mathbf{p}-\mathbf{p}_i(t))\\ \tilde{f}(\mathbf{r},\mathbf{p},t)&=w\sum_{i=1}^{\tilde{A}}\delta^3(\mathbf{r}-\tilde{\mathbf{r}}_i(t))\ \delta^3(\mathbf{p}-\tilde{\mathbf{p}}_i(t)) \end{aligned} \tag{3}$$

where $A$ and $\tilde{A}$ are the number of test-particles and anti-particles respectively, while $w$ is a normalization factor which is related to the number $B$ of the real quarks of system by

$$\int d^3rd^3p\ (f(\mathbf{r},\mathbf{p},t)-\tilde{f}(\mathbf{r},\mathbf{p},t))=w(A-\tilde{A})\equiv B\ . \tag{4}$$

To satisfy Eq.(1), the position and momentum of the $i$-th numerical quark (anti-quark) should obey the relativistic Hamilton's equations. Namely,

$$\begin{aligned} \dot{\mathbf{r}}_i&=\mathbf{p}_i/E_i\quad,\quad\dot{\mathbf{p}}_i=-\vec{\nabla}_rE_i\quad,\quad E_i=\sqrt{\mathbf{p}_i^2(t)+M_c^2(\mathbf{r}_i,t)}\\ \dot{\tilde{\mathbf{r}}}_i&=\tilde{\mathbf{p}}_i/\tilde{E}_i\quad,\quad\dot{\tilde{\mathbf{p}}}_i=-\vec{\nabla}_r\tilde{E}_i\quad,\quad\tilde{E}_i=\sqrt{\tilde{\mathbf{p}}_i^2(t)+M_c^2(\tilde{\mathbf{r}}_i,t)} \end{aligned} \tag{5}$$

where dots indicate time differentiations. We have investigated numerically the equations of motion (5) together with the gap equation (2) in a self-consistent manner. We have performed our calculations on a 3 dimensions 41 point lattice with a $a = 0.25$ fm mesh size. The initial conditions are taken as follows : we have considered $A$ test-quarks and $\tilde{A}$ test anti-quarks (see below for the numerical values) distributed randomly within a sphere of radius $r_0 = 1.05$ fm. The position of each test particle is assigned by choosing:

$$r = r_0(x_1)^{1/3} \quad ; \quad \cos(\theta) = 1 - 2x_2 \quad ; \quad \phi = 2\pi x_3 \tag{6}$$

where $x_1, x_2$ and $x_3$ are random numbers between 0 and 1. The Cartesian coordinates of the test particle are $r\cos\phi\sin\theta$, $r\sin\phi\sin\theta$ and $r\cos\theta$. The momentum of the test particles are choosen randomly according to a Fermi distribution. It means that initially the system is in equilibrium. The coordinates of each test-particle momentum are $[p_x = p\cos\phi_p\sin\theta_p,\ p_y = p\sin\phi_p\sin\theta_p,\ p_z = p\cos\theta_p]$ where the angles are choosen randomly as above while $p$ is determined from

$$\int_0^p p'^2 dp'\ (1 + e^{\frac{E_{p'}-\mu}{T}})^{-1}\ /\ \int_0^\Lambda p'^2 dp'\ (1 + e^{\frac{E_{p'}-\mu}{T}})^{-1} = x_1 \tag{7}$$

In the last equation $x_1$ is a random number, $E_{p'} = \sqrt{p'^2 + M_c^2(0)}$ where $M_c(0)$ is the space-independent solution of the gap equation (2) for a fermi distribution, $T$ represents the temperature, $\mu$ the chemical potential and $\Lambda$ the cutoff. The minus sign in the exponentials should be replaced by a plus sign in the case of a test anti-particle. In this work we have taken initial quark and energy densities of $\rho_0 = B/\frac{4}{3}\pi r_0^3 = 1.87$ fm$^{-3}$ (which leads to $B = 9$ for $r_0 = 1.05$ fm ), and $e_0 = 1.57$ Gev fm$^{-3}$ respectively. These values correspond, via the definition of quark and energy densities in the presence of a cutoff

$$\begin{aligned} \rho_0 &= \frac{2N_cN_f}{(2\pi^3)}\int^\Lambda d^3p\left(\frac{1}{1+e^{(E_p-\mu)/T}} - \frac{1}{1+e^{(E_p+\mu)/T}}\right) \\ e_0 &= \frac{2N_cN_f}{(2\pi^3)}\int^\Lambda d^3p\ E_p\left(\frac{1}{1+e^{(E_p-\mu)/T}} + \frac{1}{1+e^{(E_p+\mu)/T}}\right) \end{aligned} \tag{8}$$

together with the gap equation (2), to the following values for the temperature, the chemical potential and the constituent quark mass respectively: $T = 240$ MeV, $\mu = 200$ MeV and $M_c(0) = 33.5$ MeV. The numerical values of $A$ and $\tilde{A}$ for the test-quarks and test anti-quarks respectively have been fixed by fitting, at $t = 0$, to $\rho_0$ and $e_0$ given above with the result: $A = 2630$ and $\tilde{A} = 630$. According to Eq. (4) the difference $A - \tilde{A}$ determines how many test (anti) quarks one employs for a physical (anti) quark [10]. At this stage we want to stress that the well known expressions of the quark and energy densities in the case of a fermi distribution with a vanishingly small quark mass:

$$\begin{aligned} \rho_0 &= N_cN_f\mu(\pi^2 + (\beta\mu)^2)/3\pi^2\beta^2 \\ e_0 &= N_cN_f(21\zeta(4) + 6(\beta\mu)^2\zeta(2) + \tfrac{1}{2}(\beta\mu)^4)/2\pi^2\beta^4\ , \end{aligned} \tag{9}$$

lead to $T = 1/\beta = 185$ MeV and $\mu = 190$ MeV. The difference between these values and those given above is due to the cutoff. To be consistent we have considered in this work the quantities calculated with the cutoff $\Lambda$ (c.f., Eqs. (8) ).

For the numerical implementation of our equations, the evolution of the $i$-th test particle or test anti-particle (c.f., Eqs(5)) is represented by the following algorithm

$$\begin{aligned} \mathbf{p}_i(t+\delta t) &= \mathbf{p}_i(t-\delta t) - 2\ \delta t(\ \vec{\nabla} M_c(\mathbf{r}_i,t))\ M_c(\mathbf{r}_i,t)/E_i(t) \\ \mathbf{r}_i(t+\delta t) &= \mathbf{r}_i(t-\delta t) + 2\ \delta t\ \mathbf{p}_i(t)/E_i(t) \end{aligned} \tag{10}$$

where $\delta t = 0.05$ fm/c is the mesh in time. At the time $t$, we determine the constituent mass $M_c(cell,t)$ (2) at each cell of size $a$ of the space lattice ($a$ being the space mesh-size) according to

$$M_c(cell,t) = m + g^2 M_c(cell,t)\ (\frac{3}{2\pi^3} I_\Lambda(t) - \frac{w}{a^3}\sum_{i=1}^{A_{cell}} \frac{1}{E_i(t)} - \frac{w}{a^3}\sum_{i=1}^{\tilde{A}_{cell}} \frac{1}{\tilde{E}_i(t)}) \tag{11}$$

where $A_{cell}$ ($\tilde{A}_{cell}$) are all the test particles (anti-particles) which are inside the considered cell, and $I_\Lambda = \pi M_c^2(\sinh(2\alpha) - 2\alpha)$ with $\alpha = \log[\frac{\Lambda}{M_c} + \sqrt{\frac{\Lambda^2}{M_c^2}+1}\ ]$. In order to test the accuracy of our numerical method, we have checked the energy conservation. We have found that the total energy of the system changes by less than 5 percent for 40 iterations in time with a 0.05 fm/c mesh-time.

Before we come to the numerical results it is instructive to discuss our expectations using a simplified model. Let us assume for the moment that we have a freely expanding nonrelativistic plasma at a temperature T which is initially confined in a sphere in coordinate space. The (Maxwell) velocity distribution $dN/dv$ is zero for $v = 0$ and has a pronounced peak around $\bar{v} = \sqrt{2T/m}$. Without any interaction the many particles with about that velocity stay together, i.e. the maximum of the density move with $\bar{v}$ outwards. For radii considerably larger or smaller than $r(t) = \bar{v}t, (t \gg 0)$ the density is lower. Thus the particles are concentrated in a shell. The density in that shell decreases proportional to $r^{-2}(t)$.

Now, what happens if we switch on the interaction. The interaction acts like a repulsive momentum dependent potential $V = \alpha(\rho) \cdot g(p)$ where $\alpha$ increases with decreasing density. Since the potential is more repulsive in the low density region as compared to the high density region there is a force which accelerates the particles towards high density. Thus the high density zone can maintain a higher density as compared to the free streaming on expense of the surrounding lower density regions. Finally it is energetically favourable to give up the isotropy in $\phi$ and $\theta$ and to build local fluctuations (droplets) of high density which are separated by regions of lower density. However, this is beyond the limits of the predictive power of our one body theory even if we observe these fluctuations due to the finite number of test particles.

We come now to the numerical results. Fig.1 shows the behaviour of the constituent quark mass $M_c$ as a function of the radius for different times. For high density, i.e. in the plasma phase, $M_c$ is equal to the bare mass of the quark. At zero density, after the phase transition has taken place, $M_c$ is equal to the constituent quark mass of $\approx 300$ MeV. Initially we see that all particles have a mass of 33.5 MeV which can be considered as the bare mass. The fact that it is not exactly equal to 4 MeV is due to the finite density. Hence the system is in the quark phase. Then the system expands and the mass of the particles close to the surface comes close to the constituent quark mass. At $t = 1$fm/c we see that the particles have left the center, and therefore the mass has increased also there to the constituent quark level. We have now an expanding *shell*. Before and after the shell we have constituent quarks while in the interior we have the plasma. During the expansion the peak density decreases and therefore $M_c$ increases. The plasma in the interior of the shell starts to make the phase transition. This phase transition is almost accomplished after 1.9 fm/c.

Hence the result of the numerical calculations follows qualitatively the consideration explained above. The following scenario of the phase transition emerges. If initially a plasma is created in a heavy ion collisions, it is, due to geometry, confined in a limited space region. The high temperature of the plasma leads to a fast expansion of the system. The phase transition takes place while the system expands. There is a density front traveling outwards. The density is fed by the interaction of the quarks which pulls the quarks into the high density region but it also decreases with about $r^{-2}(t)$ due to the expansion. Most probably then the system gives up the spherical isotropy and forms quark droplets. However, they will not be stable but will hadronize. Obviously this scenario is quite different from that expected for a system which keeps global thermal equilibration during the expansion.

Is this phase transition specific for the NJL Lagrangian or genuine for a phase transition in an expanding system? A final answer is not at hand yet but there exists another phase transition in nuclear physics which has been investigated in detail recently [11] : The liquid (fragments)-gas (nucleons) transition during the expansion of a fireball created at much lower energy ( $E_{\rm beam} \sim 100$ MeV/N) in heavy ion reactions. There, for a density smaller than the normal nuclear matter density, we have as well the situation that the potential is more attractive at higher density as compared to lower density. In the expansion of this system a similar droplet formation has been found in Ref. [11] if one describes the expansion by a 1 - body transport theory as done here. Thus, there is evidence that our results are rather general. More realistic $N$-body calculations confirm this general structure and show that indeed many droplets are formed.

Calculations including collision terms are under way. Recent calculations of the hadronization cross section of a quark plasma [12] offers as well the possibility to model the chiral phase transition

itself. The results will be reported in a forthcoming publication [8].

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## Figure captions

**FIG. 1.** The constituent quark mass $M_c$ (c.f., Eqs. (2 ,11) ) as a function of the radius for different times. The initial temperature and density of the system are $T = 240$ MeV and $\rho_0 = 1.87$ fm$^{-3}$ respectively.

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